NUCLEAR FISSION AND ITS APPLICATION TO JET PROPULSION ENGINES

JOHN PATTON RIMER

Library
U. S. Naval Postgraduate School
Monterey, California

Mont 214
8854

Library
U. S. Naval Postgraduate School
Annapolis, Md.

NUCLEAR FISSION AND ITS APPLICATION TO JET PROPULSION ENGINES

By

John Patton Rimer
Lieutenant, U.S.N.

Submitted to the Faculty of Rensselaer Polytechnic Institute in Partial Fulfillment of the Requirements for the Degree of Master of Science

June 1947
Troy, New York

Thesis
R55

Acknowledgment

The writer wishes to express his appreciation of the helpful suggestions of Professor Neil P. Bailey and Dr. Herta R. Leng of Rensselaer Polytechnic Institute. He also wishes to extend his thanks to Mr. E. R. Prentice, General Electric Company, Schenectady, New York, for information concerning the practical utilization of atomic energy.

7557

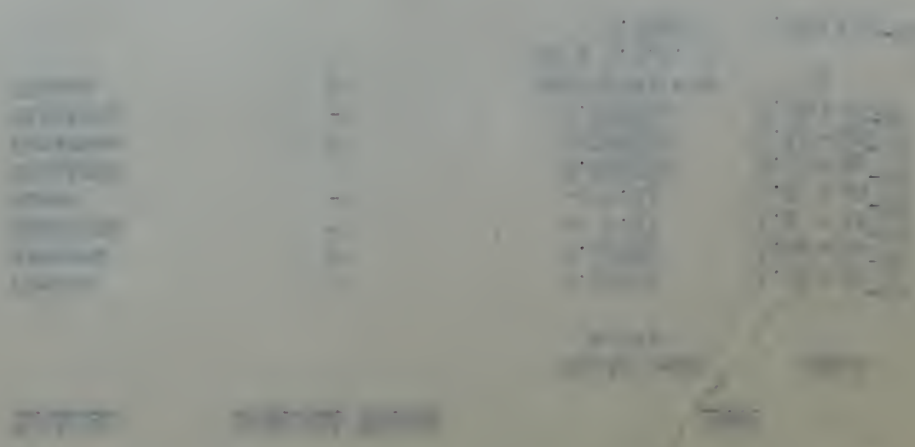
Table of Contents

Chapter I: **The Nuclear Fission Process**

It is just over fifty years ago that Henri Becquerel made his famous discovery of radioactivity in uranium salts. This was followed shortly by independent observations by Schmidt, Curie, and others of similar radiations from other sources.

The fact of radioactivity has led directly to a reconsideration of the fundamental particles of matter. If by "fundamental particle" is meant an elementary unit that under no condition can be transformed into one or more other particles, then no such particle exists. At various times it has seemed certain that such entities as protons and electrons might be so regarded. On further investigation it must be concluded that within a nucleus, or on suitable collision, protons transform into other particles, and electrons form from and convert into photons. However, there are certain particles, which although transmutable among themselves, seem to serve as the building units for more complicated matter. These particles are listed below:

Particle	Electric Charge	Mass	
		atomic mass units	grams
Proton	+	1.00758	1.66×10^{-24}
Neutron	0	1.00893	1.66×10^{-24}
Mesotron	+	$\sim$ 0.11	1.8×10^{-25}
Meson	-	$\sim$ 0.11	1.8×10^{-25}
Positron	+	0.000548	9.19×10^{-28}
Neutrino	0	0.000548	9.19×10^{-28}
Electron	-	0.000548	9.19×10^{-28}
Photon	0	rest mass = 0	0
		at 1 M.E.V. = 0.00107	1.762×10^{-28}



It should also be remembered that the radiations called alpha, beta and gamma rays are helium nuclei, high speed electrons, and very penetrating x-rays respectively. The neutron is of especial importance in the nuclear fission process. Therefore, its characteristics will be investigated more closely.

The Neutron:

In 1930 W. Bothe and H. Becker in Germany found that if the very energetic natural alpha particles from polonium fell on certain of the light elements, particularly boron and lithium, an unusually penetrating radiation was produced. This radiation was assumed to be very penetrating gamma rays. I. Curie-Joliot and F. Joliot proceeding on the same assumption measured the absorption coefficients of the supposed gamma rays in lead. Their observations gave linear absorption coefficients of 0.147 and 0.227 cm^{-1} for beryllium and boron respectively. It was not then known that such low values were impossible for any substance because of absorption due to electron pair formation. They further observed that if this unknown radiation fell on paraffin or any other hydrogen-containing compound it ejected protons of very high energy. These protons had a range up to 26 cm. in air, making it necessary to postulate that the energy of the incident gamma quanta was about 55 M.E.V., over twice the energy measured by the Joliot's.

J. Chadwick in England, 1932, confirmed the experimental results and finally came to the conclusion that the only justification of the high energy protons was by assuming that the radiation from

beryllium and boron consisted of energetic neutral particles. These had arisen from nuclear disintegration caused by the alpha particles and could penetrate matter with little absorption because of their lack of electric charge. These particles he called "neutrons".

The very property of lack of charge characterizing the neutron delayed its discovery by making direct observation impossible. This characteristic also makes neutrons very penetrating and thus important agents in nuclear change. Charged particles and electromagnetic radiations lose energy in passing through matter. They exert electric forces which ionize atoms of the materials through which they pass. The energy produced by ionization equals the energy lost by deceleration or absorption. The neutron is affected only by a very short range force that becomes significant when the neutron passes very close to an atomic nucleus. This force is sometimes called "super-gravitational", and is the force that holds the nucleus together in spite of the mutual repulsion of the positively charged protons contained.

Since the neutron carries no net electric charge it cannot be deflected in either a magnetic or electric field. Hence conventional methods, such as mass-spectroscopy, cannot be used to measure its mass. Only indirect methods may be used. The masses of the deuteron (heavy hydrogen nucleus) and the proton may be determined separately in a mass-spectrograph to be 2.014174 and 1.007582 atomic mass units respectively. The threshold energy for photoelectric disintegration of the deuteron is about 2.18 M.E.V. Einstein's equation for the conversion of mass to

energy,

$$E = m c^2$$

must be used; where E = energy, m = mass, c = velocity of light, all expressed in compatible units. From this we find that an energy of 2.18 M.E.V. corresponds to 0.002335 atomic mass units. Then an energy balance for the deuteron disintegration in terms of mass units is:

deuteron + photon $\longrightarrow$ proton + neutron

$$_1H^2 + h\nu \longrightarrow {}_1H^1 + {}_0n^1$$

2.014174 + 0.002335 = 1.007582 + mass of neutron

giving the mass of the neutron equal to 1.00893 a.m.u. In the equation above numerical subscripts refer to electric charge, superscripts to mass number (atomic mass units to nearest whole number).

It was mentioned above that the neutron can experience a force only when it comes within extremely close range of an atomic nucleus. The interaction with the nucleus may be regarded as a collision, which may be either elastic or inelastic. In an elastic collision the ordinary conservation of energy and momentum laws apply. A portion of the energy of the impinging neutron is transferred to the struck nucleus. If the target nucleus is a heavy one such as lead, the energy loss by the neutron is very small. The limiting case would be the rebound of a neutron from a hypothetical wall with no energy loss. Conversely, if the nucleus of a light atom, with a mass of the same order of magnitude

as that of the neutron, is struck, the energy of a high speed neutron is reduced considerably at each collision. Thus a given thickness of material containing light atoms, such as water, hydro-carbons, etc., will more effectively reduce the intensity of a beam of energetic neutrons than the same thickness of lead. As the energy of the neutron decreases, it approaches an energy of 3/2 k T corresponding to that of a particle in thermal equilibrium with its surroundings. "k" is the Boltzmann gas constant per particle, 1.37×10^{-16} ergs per degree Centigrade, and "T" is the temperature on the absolute Kelvin scale. A temperature of 300°K corresponds to an energy of about 0.038 electron volts. A neutron in this energy range is usually referred to as a "thermal" neutron.

The case of inelastic collision occurs when the impinging neutron enters the struck nucleus and combines with the other particles to form a new isotope of the target element. The nucleus is usually left in an excited state, with any excess energy being carried off by the emission of radiation. This process, called "capture", often leads to new, unstable or radioactive isotopes of either the bombarded element or one near it in the periodic table. The reaction occurring at impact may consist in the emission from the compound nucleus of any one of the following: (1) gamma ray, (2) proton, (3) alpha particle, (4) two neutrons, (5) three particles, and (6) fission of the nucleus. There is also the case of the original neutron being reemitted at reduced energy with part of the original energy going to excite the nucleus and put it at a higher energy level.

In the neutron-gamma ray reaction the product nucleus retains the captured neutron and any excess energy is carried off by the emission of a gamma ray. Therefore, the nucleus that is formed is larger in atomic weight by one, than the target nucleus. If this isotope does not exist in nature, it is certain to be radioactive. This is a very likely reaction when the energy of the incident neutron is low. Any release of energy because of the mass difference between the initial and final products of the reaction, as well as the energy of the impinging neutron, will be carried away by the gamma ray. A systematic survey of the elements of the periodic table for this reaction was carried out by Fermi and his associates in Rome.[1] * They began with oxygen and found 42 different radioactive products in their first examination. This list has since been greatly extended.

The uncharged neutron can enter the nucleus with low energy; also, it can leave without having to overcome a barrier energy. For a charged particle to escape the nucleus, a potential barrier of force tending to keep the particle inside must be overcome. The only charged particle that exists for an appreciable length of time in the nucleus is the proton. The value of the barrier potential is an oscillating quantity. While a proton of energy less than the average barrier potential may escape if it arrives at the surface at a favorable moment, the probability of escape increases as the energy of the particle until the energy equals the maximum barrier potential, at which point a saturation is obtained. When a neutron enters a nucleus it may strike

---

* Numbered superscripts refer to references of corresponding number listed in the bibliography.

a proton, imparting sufficient energy for the proton to overcome the barrier potential and escape from the nucleus. This process of proton emission may be regarded as in competition with the straight neutron capture given above and increases in importance for high energy impinging neutrons and for elements of low atomic number.

The reaction for any element X may be shown as:

$$ {}_{Z}X^{A} + {}_{0}n^{1} \longrightarrow {}_{Z-1}X^{A} + {}_{1}H^{1} + \text{energy} $$

where A = atomic mass number, Z = charge of nucleus.

The resultant energy together with any incident kinetic energy is shared by the resultant products. The product nucleus usually emits a beta quantum and reverts to the original isotope.

When some of the light elements are bombarded with energetic neutrons they emit alpha particles (helium nuclei). A typical reaction is shown for the case of boron:

$$ {}_{5}B^{10} + {}_{0}n^{1} \longrightarrow {}_{3}Li^{7} + {}_{2}He^{4} + \text{energy} $$

In this reaction there is a loss of 0.00322 a.m.u., which gives a resultant energy of 3.0 M.E.V.

An inspection of a table of isotopes reveals that the mass difference between adjacent isotopes of any of the heavier elements is almost exactly one unit. The mass of the neutron is 1.00893 a.m.u., and these nuclei differ in composition by one neutron. Therefore, there is a mass discrepancy of about 0.00893 a.m.u. which is equivalent by

Einstein's conversion to the energy that must be added to convert the heavier isotope into the lighter one plus a neutron. This energy is around 9 ± 2 M.E.V. Thus if a neutron of 10 or more M.E.V. energy enters a nucleus, two neutrons may be simultaneously ejected (the incident neutron at reduced energy plus a neutron from the nucleus). This process gives an isotope of atomic weight one less than the original nucleus. If the product isotope does not exist naturally, it is almost certain to be radioactive and an emitter of positrons. In general:

$$ {}_{Z}X^{A} + {}_{0}n^{1} + \text{Energy} \longrightarrow {}_{Z}X^{A-1} + {}_{0}n^{1} + {}_{0}n^{1} + \text{energy} $$

or, for a typical case, phosphorus 31:

$$ {}_{15}P^{31} + {}_{0}n^{1} + W \longrightarrow {}_{15}P^{30} + {}_{0}n^{1} + {}_{0}n^{1} + w $$

a mass-energy balance in terms of mass gives:

$$ 30.9839 + 1.00893 + W = 29.9882 + 2(1.00893) + w $$

w equals the resultant energy. W equals the original energy and must be greater than w by an amount equal to the change in total mass of reactants and products. The mass discrepancy is 0.0132 a.m.u. Therefore, W must be greater than w by 0.0132 a.m.u., or 12.3 M.E.V.

An extensive survey was made[2] in which most of the elements of the periodic table were bombarded by 20 M.E.V. neutrons. A total of 115 radioactivities were observed, of which about one third were emitters of electrons and were identified with the same activities produced by

Fermi in neutron capture. The remainder were positron emitters of the type of the products in the reaction described here.

The detachment of a neutron from the nucleus requires expenditure of 8 - 10 M.E.V. of energy. When a nucleus is struck by a neutron of 20 M.E.V. or more there is a possibility that the excited nucleus will give off three neutrons or equivalent particles in disintegration. The probability of such an occurrence would increase with increasing incident neutron energy. This reaction has been produced experimentally by bombarding sulfur with very high energy neutrons.[3] The resultant isotope obtained was phosphorus in the following reaction:

$$_{16}S^{32} + {}_{0}n^{1} + W \longrightarrow {}_{15}P^{30} + {}_{0}n^{1} + {}_{0}n^{1} + {}_{1}H^{1} + w$$

The mass discrepancy is equivalent to a difference (W - w) of 20.5 M.E.V. An alternate reaction would be the emission of three neutrons and a positron.

There is another type of reaction that will be discussed in more detail below. When very heavy nuclei are made unstable by the addition of a neutron they may break up into two parts of comparable mass instead of emitting a small particle. This process, fission, has been found to be possible for all elements of atomic number ninety or more. The minimum energy of the incident neutron ranges from a few hundredths of an electron volt to 1.5 million electron volts, depending on the isotope and element involved. The energy release of these elements, all radioactive, is the same as it would be over the long period of time required for natural disintegration; it is of the order of 190 M.E.V.

The Fission Reaction:

Fermi, in his survey of the sensitivity of the elements to neutron capture[4], found several new radioactivities for uranium. These substances decayed with the emission of beta particles which meant that the product must have an atomic number (equals nuclear charge) of 93.

$$_{92}U^{238} + {}_{0}n^{1} \longrightarrow {}_{92}U^{239} \longrightarrow {}_{93}X^{239} + {}_{-1}e^{0}$$

Examination of the periodic table showed that the new element should follow the chemistry of manganese. A separation based on this reasoning was made which yielded a radioactive precipitate. Further separations based on the chemistry of iron, rhodium, and platinum, gave radioactive products indicating the presence of elements of atomic number 94, 95, and 96, respectively. Thus elements beyond uranium, which had been considered the end of the periodic table, were found. These were called "transuranic" elements and have been named neptunium, plutonium, americium, and curium, in order up to atomic number 96 of the heaviest one so far observed.

In a more detailed investigation of the products of neutron bombarded uranium, activities were found in barium and lanthanum type precipitates by Hahn, and I. Curie. These results did not gibe with the assumption of existence of any transuranic elements. Further tests revealed that radioactive barium and lanthanum were actually present, not other chemically similar elements. This meant that by some unknown process elements far removed in the periodic table from the target

element were being produced. All previous work with nuclear transformations had yielded products one or two places to the right or left of the original element in the periodic chart. Hahn and Strassmann found among the radioactivities observed one of krypton of atomic number 36. When this is combined with the atomic number of barium, 56, 92 is obtained corresponding to the atomic number of uranium. Reluctant "to contradict the previous lessons in nuclear physics" they yet advanced the idea that fission of the uranium nucleus into two parts of comparable mass had occurred.

As soon as the ice was broken, confirming tests and reinterpretations of previous results were made by a number of physicists. The first publication by Hahn and Strassmann had come on January 16, 1939 in "Naturwissenschaften". By the end of February, over forty papers appeared on uranium fission. The phenomenon attracted such wide interest because of the extremely large amount of energy released from the fission of each nucleus.

Observation of a large number of fission particles and their energy was made by McMillan, Hafstad, Dunning, and others, using foil penetration, ionization chamber, and cloud chamber tracks as a means of measuring energy. Compilation of results showed that the number of particles in a particular energy interval reached a maximum at two points, one at 64 M.E.V. and the other at 97 M.E.V. This correlated with probability theory on the splitting of a small mass into two parts. Division into two equal parts would occur rarely. The sum of the energies of the two maxima is 161 M.E.V.

Henderson[5] made a sensitive calorimetric determination of the total average energy per fission. He concluded that the total energy produced by a uranium fission was about 185 M.E.V. This may be compared with the 2 to 5 electron volts released per molecule of combustion product formed in ordinary burning. For a more practical example, one pound of U^{235} contains 11.6×10^{23} nuclei; an average energy release of 185 M.E.V. per particle gives:

$$(11.6 \times 10^{23})\ (185\ \text{M.E.V.}) = 2.14 \times 10^{26}\ \text{M.E.V.} = 34.4 \times 10^{19}\ \text{ergs}$$

Energy release per lb. = 9,550,000 kilowatt hours

From Cork's "Table of Isotopes" [6] the natural isotopes of uranium are 234 (0.006%), 235 (0.71%), 238 (99.28%). Nier and Dunning, at Columbia, demonstrated in 1940 that U^{235} would fission by slow neutron capture. To produce this thermal neutron induced fission a very large probability of fission for a given neutron density was observed. To produce fission in a U^{238} nucleus a neutron energy of at least 1.5 M.E.V. is required and the probability of the fission occurring is small. Thorium and protoactinium will undergo fission, but again neutrons of high energies, 1.1 M.E.V. and 1.0 M.E.V., are required.

When it was known that thermal neutrons would induce fission in U^{235}, it became of interest to find if any slow neutrons resulted from the reaction. This would permit a sustained energy release provided only that U^{235} nuclei were in the neighborhood to absorb any neutrons produced. Roberts and his associates[7] showed that from one to

three neutrons were set free at fission of a uranium nucleus. This is reasonable when the following reaction is considered:

$$_{92}U^{235} + {}_{0}n^{1} \longrightarrow {}_{92}U^{236} \longrightarrow {}_{46}Pd^{118} + \text{energy}$$

The heaviest natural isotope of palladium has an atomic mass number of 110. Therefore, it would be expected that the product nucleus would be highly unstable and would tend toward a stable state by emission of neutrons, beta particles, or both. In addition to the neutrons given off during bombardment, a delayed neutron emission was observed. The neutrons emitted at fission were not of thermal energy, as expected, but of energies on the order of 1 M.E.V. The conditions found to exist were still favorable for the chain reaction to take place provided the emitted neutrons were absorbed by fission processes.

From Einstein's equation for the conversion of mass to energy, the energy released by the division of a nucleus into smaller parts may be expressed as

$$\Delta E = (M_o - \Sigma M_1)\, c^2$$

where M_o is the mass of the original nucleus, M_1 are the masses of the product fragments, c is the velocity of light expressed in the same system of units as the masses. This expression would provide a simple way of calculating the energy release from a uranium fission were it not for the fact that the product nuclei are so unstable that no measurements of their masses have been obtained. (See previous example of uranium to

palladium fission.) This instability is occasioned by the abnormal ratio of mass to charge formed by the division of a heavy nucleus into two parts of comparable size.

Gamow has obtained a semi-empirical expression for the difference in mass neglecting energy fluctuations due to odd-even alternations of atomic mass and number, and the finer details of nuclear binding:

$$\text{mass change} = {}_{Z}X^{A} - {}_{Z_A}X^{A} = \frac{1}{2} B_A (Z - Z_A)^2$$

Z = charge number of fragment

Z_A = quantity which in general is not an integer and may be found in Table 1

A = masses of nuclei X

B_A = a quantity obtained from Table 1 which is not determinable by experimental means.

Table 1

Values of Z_A, B_A, δ_A estimated[8] for various values of nuclear mass A. B_A and δ_A are in million electron volts, A in atomic mass units, Z_A in units of charge.

A	Z_A	B_A	δ_A
50	23.0	3.5	2.8
60	27.5	3.3	2.8
70	31.2	2.5	2.7
80	35.0	2.2	2.7
90	39.4	2.0	2.7
100	44.0	2.0	2.6
110	47.7	1.7	2.4
120	50.8	1.5	2.1
130	53.9	1.3	1.9
140	58.0	1.2	1.8
150	62.5	1.2	1.5
160	65.4	1.1	1.3
170	69.1	1.1	1.2
180	72.9	1.0	1.2
190	76.4	1.0	1.1
200	80.0	0.95	1.1
210	83.5	0.92	1.1
220	87.0	0.88	1.1
230	90.6	0.86	1.0
240	93.9	0.83	1.0

The value of B_A may be obtained in the following manner. First it is assumed that the energies of nuclei with a mass A will vary approximately with charge Z as given below:

$$\text{energy of } {}_Z X^A = c_A + \frac{1}{2} B'_A (Z - \tfrac{1}{2} A)^2 + (Z - \tfrac{1}{2} A)(M_p - M_n) + 3 Z^2 e^2 / 5 r_o A^{1/3}$$

7867

The first term is constant, the second term gives the comparative masses of the various isobars (nuclei of equal mass but different charge) neglecting the influence of the difference $(M_p - M_n)$ of the total proton and neutron masses, which is included in the third term, and of the pure electrostatic energy given by the fourth term. The symbol e is the unit of charge, it equals the charge of the electron. In the last term the assumption is made that the effective radius of the nucleus is equal to $r_o A^{1/3}$, where the value of r_o is approximately 1.48×10^{-13} from the theory of alpha ray disintegration. Combining the relative mass values from Gamow's equation and the one above:

$$B_A = B'_A + \frac{6 e^2}{5 r_o A^{1/3}} = (M_p - M_n + 3 A^{2/3} e^2/5 r_o) / (\tfrac{1}{2} A - Z_A)$$

It is possible to estimate the mass of the nucleus of charge Z and mass number A on the above basis with the help of the packing fraction of the known nuclei:

$$M_{(Z,A)} = A (1 + f_A) + \tfrac{1}{2} B_A (Z - Z_A)^2 + \begin{cases} +\,0 \text{ if A odd} \\ -\,\tfrac{1}{2} \delta_A \text{ if A even Z even} \\ +\,\tfrac{1}{2} \delta_A \text{ if A even Z odd} \end{cases}$$

where $M_{(Z,A)}$ is the mass of a nucleus of atomic number Z and mass number A; f_A is the average value of packing fraction over a small region of atomic weights. The last term corrects for the typical variation in binding energy among nuclei according to the odd or even character of their neutron and proton numbers.

For lower masses it is obtained by consideration of the energy release involved in beta ray emission or absorption by a nucleus:

$$E_B = B_A \left\{ \left| Z - Z_A \right| - \frac{1}{2} \right\} \begin{array}{l} + 0 \text{ if A odd} \\ - \delta_A \text{ if A even Z even} \\ + \delta_A \text{ if A even Z odd} \end{array}$$

This equation, obtained from the previous one for nuclear mass, permits estimation of δ_A by an examination of the stability of isobars of even nuclei. If an even-even nucleus is stable then δ_A is greater than the first term in the equation, and conversely if the nucleus is unstable. For nuclei of medium masses δ_A is bracketed quite closely. For the region of very high atomic masses, δ_A can be estimated directly from the difference in energy release of successive beta ray transformations such as:

$$_{90}Th^{234} \xrightarrow{\beta} {}_{91}Pa^{234} \xrightarrow{\beta} {}_{92}U^{234}$$

$$_{88}Ra^{226} \xrightarrow{\beta} {}_{89}Ac^{226} \xrightarrow{\beta} {}_{90}Th^{226}$$

From the foregoing, using available measurements of nuclear masses, estimates were obtained by Bohr for the energy release on division of a nucleus into two approximately equal parts for various elements. He calculated that U^{238}, upon absorption of a neutron and fission into palladium, would give up 200 M.E.V. at fission and about 31 M.E.V. additional could be obtained from the unstable palladium nuclei.

This checks quite well with previously mentioned experimental values of 185 - 190 M.E.V. fission energy release.

A further interesting result of Bohr's calculations is that if nuclei break into more than two comparable parts, a process as yet unknown, in many cases there would be an accompanying energy release. Nuclei of mass number greater than 110 break into three parts exothermally. If uranium breaks into three parts the fission energy release would be slightly higher than on normal fission. For four way division the energy output decreases to about 150 M.E.V. The energy further declines with higher division until the reaction becomes endothermic by the time fifteen-way division is reached.

Nuclei are energetically stable with respect to division below atomic masses of one hundred; that is, they absorb energy in the fission process. For higher masses instability sets in primarily because the decrease in electrostatic energy associated with the separation overcompensates the desaturation of short range forces consequent on the greater exposed nuclear surface.

The energy output involved on fission of the unstable U^{239} nucleus into two fragments of any given charge and mass numbers is nearly a maximum for a long range of product masses. However, for a given fragment size (mass) there is only a small range of charge numbers corresponding to an energy release near the maximum value. It has been found that the most favorable division of uranium, from an energy point of view, produces fragments separated from their corresponding

stable nuclei by an amount equivalent to the change in nuclear charge associated with the emission of from three to six beta particles. The amount of energy released in beta ray transformations following fission, E_p, may be estimated from the equation given above with the aid of Table 1.

The amount of energy available for beta ray emission from typical product nuclei doesn't conflict with the stability of these nuclei to spontaneous neutron emission. This may be seen from the fact that the energy change accompanying an increase of nuclear charge by one unit is the difference between the binding energy of a neutron and a proton, plus the mass discrepancy between a proton and a neutron. The binding energy of a neutron in typical unstable product nuclei in the region of greatest energy release can be estimated, from the last equation for nuclear mass above, and gives from 5.2 to 6.4 M.E.V. for the natural isotopes of uranium in fission form. For certain reactions such as

$$_{46}Pd^{125} \xrightarrow{\beta} {}_{47}Ag^{125} \xrightarrow{\beta} {}_{48}Cd^{125}$$

$$_{49}In^{130} \xrightarrow{\beta} {}_{50}Sn^{130}$$

$$_{52}Te^{140} \xrightarrow{\beta} {}_{53}I^{140} \xrightarrow{\beta} {}_{54}Xe^{140}$$

The energy releasable by beta ray transformations is greater than the neutron binding energy. It will be shown later that this offers a reasonable explanation for the delayed neutron emission accompanying the fission process.

The Liquid Drop Analogy of the Nucleus

The nucleus possesses forces not electrical or magnetic in nature which are best understood by likening them to the cohesive and surface tension forces in a liquid droplet. In order to obtain a workable method of handling nuclear phenomena, Bohr and Wheeler made an analysis[8] of the fission process using the mechanics of a liquid droplet as a basis. While not completely accurate, it is generally conceded to be the best system available to date for describing the state of the nucleus as it undergoes the fission reaction, or is influenced by neutrons in associated reactions.

By predication of a liquid drop model of the nucleus, the excitation energy must be expected to give rise to modes of motion of the nuclear matter similar to the oscillations of a fluid sphere under the influence of surface tension. The high nuclear charge of the heavy nuclei, however, will counteract to a large extent the restoring forces due to the short range attractions responsible for the surface tension of nuclear matter.

The possibility of quantum mechanical tunnel effects is of interest in a consideration of nuclear stability. They make it possible for a nucleus to divide even in its stable ground state by allowing passage of a fragment through a portion of configuration space where classically the kinetic energy is negative. This corresponds to the varying value of the magnitude of the potential barrier, mentioned earlier, as compared to the fixed average value. The importance of this effect is taken into account by investigating the stability of a nucleus

for small deformations as well as for ones large enough that fission may be expected actually to occur.

Take a small arbitrary deformation of the liquid drop analogous to the nucleus such that the distance from the center to an arbitrary point on the surface with colatitude θ is changed from its original value R to the value

$$r(\theta) = R\left[1 + \alpha_0 + \alpha_2 P_2(\cos\theta) + \alpha_3 P_3(\cos\theta) + \ldots\right]$$

where the alpha's are small quantities. Then Bohr shows that the sum of the surface and electrostatic energies of the droplet has increased to:

$$E_{S+E} = 4\pi(r_0 A^{1/3})^2\, \sigma \left[1 + \frac{2\alpha_2^2}{5} + \frac{5\alpha_3^2}{7} + \ldots + (n-1)(n+2)\frac{\alpha_n^2}{2(2n+1)} + \ldots\right]$$

$$+ 3(Ze)^2/5r_0A^{1/3}\left[1 - \frac{\alpha_2^2}{5} - \frac{10\alpha_3^2}{49} - \ldots - \frac{5(n-1)\alpha_n^2}{(2n+1)^2} - \ldots\right]$$

from the ground state value if the assumption is made of an incompressible liquid drop of volume

$$\left(\frac{4\pi}{3}\right)R^3 = \left(\frac{4\pi}{3}\right) r_0^3 A$$

having surface tension σ and uniformly electrified to a charge Ze.

The coefficients of α_2^2 in the two terms of the equation for distortion energy may be combined to give:

$$\frac{8}{5}\pi r_0^2\, 0\, A\left[1-\left(\frac{Z^2}{A}\right)\left(e^2/10\left(\frac{4\pi}{3}\right)r_0^3\, 0\right)\right]$$

which gives a limiting value of $\frac{Z^2}{A}$ as the bracketed term tends to zero of

$$1-\left(\frac{Z^2}{A}\right)\left(e^2/10\left(\frac{4\pi}{3}\right)r_0^3\, 0\right)=0$$

$$\left(\frac{Z^2}{A}\right)_{\text{limiting}} = 10\left(\frac{4\pi}{3}\right)r_0^3\, 0/e^2$$

Beyond this limiting value of the ratio of charge number squared to mass number the nucleus is no longer stable for even the simplest deformations.

Numerical solution of the above may be made with the help of Feenberg's investigations.[9] He found that for best agreement with Bethe's semi-empirical formula, and experimental results:

$$r_0 \cong 1.4\times 10^{-13} \text{ cm.}$$

and $$4\pi r_0^2\, 0 \cong 14 \text{ M.E.V.}$$

Then a limiting value of Z^2/A is found which is about 17% greater than the ratio $(92)^2/238$ for uranium 238. From this it is concluded that the heavy nuclei such as uranium and thorium are quite near the limit of stability beyond which it is impossible for a nucleus to exist. This limit is the point where exact balance is obtained between the

electrostatic forces of repulsion and the short range nuclear attractive forces. It must be remembered that the value of this limit is based on semi-empirical and indirect determinations of surface energy to electrostatic energy ratio.

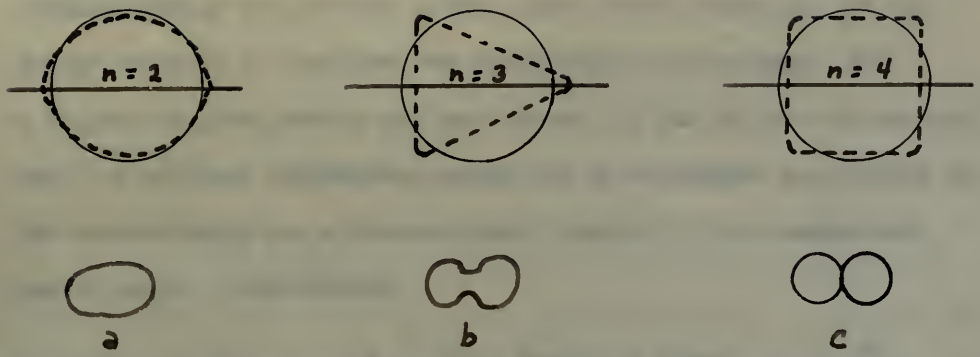
The nuclei just under the limiting ratio of charge to mass are stable with respect to very small deformations. Larger deformations, however, will tip the balance in favor of the long range repulsions as against the short range forces responsible for surface tension. Thus it will be possible for a suitably deformed nucleus to divide spontaneously.

The critical conformation of the droplet on the verge of fission is of interest. The drop will then possess a shape corresponding to unstable equilibrium and will require an amount of work that vanishes in first order terms to produce an infinitesimal displacement. (See Fig.1).

Bohr covers this point in more detail by considering the surface obtained from plotting the potential energy of an arbitrary distortion as a function of the parameters which specify its magnitude and form. Then the potential barrier hindering fission may be compared to a mountain pass running between valleys of potential. The deformation parameters corresponding to the saddle point, or high point of the pass, give the critical form of the drop. They also give the potential energy required for this distortion, which equals the critical energy for fission E_f.

Consider a continuous change in shape of a droplet from the stable sphere to two spheres of half the volume at infinitesimal

Figure 1:



The upper part of the figure[8] shows small liquid drop deformations which give characteristic oscillations of the fluid about the spherical form of stable equilibrium even when the droplet is uniformly charged. If the charge reaches the critical value of

$$\sqrt{10 \times \text{surface tension} \times \text{volume}}$$

the nucleus becomes unstable for even infinitesimal deformations of the type n = 2. A slightly smaller charge requires a finite deformation of type (a) to produce an unstable equilibrium form. The critical form changes with decreasing charge until the form (c) of two uncharged spheres an infinitesimal distance apart is obtained for the limiting case.

separation. Then, by suitable choice of the sequence of shapes, the critical energy for fission is the lowest value obtainable for the energy required to lead from one configuration to another. For a nucleus of given charge and mass number, it can be shown dimensionally that the critical deformation energy can be considered the product of the surface energy and a dimensionless function of the charge-mass number ratio. Bohr obtains:

$$E_f = 4\pi r_o^2 \, O \, A^{2/3} \, f\left[(Z^2/A) \,/\, (Z^2/A)_{\text{limiting}}\right]$$

where E_f can be determined if the shape of the nucleus in the critical state is known. This is defined by the equation for a surface in equilibrium under the action of surface tension, O, and a system of volume forces represented by a potential ϕ, with k the total normal curvature of the surface:

$$k\,O + \phi = \text{constant}$$

The practical solution of this equation for large deformations is extremely difficult. Therefore, the critical surface and dimensionless function are solved only for the special case that follows.

If the volume potential term ϕ vanishes the curvature of the unstable equilibrium surface becomes a constant. This means that the drop is divided into two spheres, since that is the only case of constant curvature unstable equilibrium. If there are no electrostatic forces to aid fission, the critical energy for fission into two

particles will equal the work done to overcome surface tension in the separation:

$$E_f = 2\left[4\pi r_o^2\, 0\, (A/2)^{2/3}\right] - 4\pi r_o^2\, 0\, A^{2/3}$$

then $f(0) = 2^{1/3} - 1 = 0.260$

If the charge of the droplet is not zero, but still small, the critical shape is not much different from that of two spheres in contact. Actually, there exists only a narrow neck of fluid connecting the two portions (see Fig. 1b) of radius r_n so as to give equilibrium. For a first approximation:

$$2\pi r_n\, 0 = (Ze/2)^2 \,/\, \left[4\, r_o^2\, (A/2)^{2/3}\right]$$

or

$$\frac{r_n}{r_o}\, A^{1/3} = 0.66\, (Z^2/A) \,/\, (Z^2/A)_{\text{limiting}}$$

Since the connecting fluid produces a second order change in energy, the critical energy for fission is calculated to the first order in charge-mass ratio. Then it is only necessary to compare the sum of surface and electrostatic energy for two spherical nuclei in contact with each other. For this Bohr obtains:

$$\begin{aligned} E_f = {} & 2\left[4\pi r_o^2\, 0\, (A/2)^{2/3}\right] - 4\pi r_o^2\, 0\, A^{2/3} \\ & + 2\left[3\, (Ze/2)^2 \,/\, 5\, r_o\, (A/2)^{1/3}\right] \\ & + (Ze/2)^2 \,/\, 2\, r_o\, (A/2)^{1/3} - 3(Ze)^2 \,/\, 5 r_o\, A^{1/3} \end{aligned}$$

Now let

$$x = (Z^2/A) \;/\; (Z^2/A)_{\text{limiting}} = \frac{(\text{charge})^2}{10 \times \text{surface tension} \times \text{volume}}$$

then, if x is a small quantity:

$$\frac{E_f}{4\pi r_0^2 \sigma_0 A^{2/3}} = f(x) = 0.260 - 0.215\, x$$

For the case of greatest immediate interest, elements of very high atomic weight, only a small deformation from spherical form is needed to reach the critical state because the value of Z^2/A is near the stable limit. The equation for E_{S+E} on page 21 gives the potential energy needed for an infinitesimal distortion as increasing with the square of the amplitude of deformation. To find a deformation corresponding to a maximum value of potential energy, a more precise method of calculation is needed.

Bohr and Wheeler presented an equation [8] for distortion energy the validity of which is now seriously questioned.[10] Further refinements in theory of the liquid drop analogy are expected to be released for publication in the near future. The difficulty in obtaining a solution is due to the fact that at least four modes of nuclear motion must be taken into account; in addition, coupling effects between various harmonics of appreciable amplitudes have a decided influence on the results. By considering the two limiting

values of critical fission energy obtained, however, they did get a semi-empirical expression for critical values in terms of x:

$$\frac{E_f}{4\pi r_o^2 \sigma A^{2/3}} = f(x) = 98\,(1-x)^3 / 135 - 11368\,(1-x)^4 / 34425 + \cdots$$

with x defined as above. With this as a basis an estimate was made for critical fission energy of uranium 238 equal to 6 M.E.V. The most reliable experimental evidence indicates a value of from 1 to 1.5 M.E.V.

From a practical point of view the fission process is irreversible. If the fission product nuclei are imagined to be toward each other without loss of energy, the electrostatic repulsion between the two nuclei will ordinarily prevent contact. The only chance for reunion of the particles would be for them to come together distorted in the same manner as at the instant of fission, and with the distortions so oriented that the two surfaces would touch at some point before the repulsive forces became too great. The union would also have to take place before any further disintegration of the product nuclei occurred. This extremely short time interval, together with the other conditions makes probability of reunion extremely small.

Fission Produced by Thermal Neutrons

A number of observers have irradiated natural uranium with thermal neutrons and found a great variety of radioactive periods arising from the fission products. The fission cross-section for

natural uranium to thermal neutrons is between 2 and 3×10^{-24} cm^2, or about twice the cross-section for radiative capture. Nier[11] found that the cross-section of U^{235} for fission absorption of thermal neutrons is about 400×10^{-24} cm. If this is considered in proportion to the amount of U^{235} present in natural uranium, 1/140, it is seen that fission with thermal neutrons is carried out almost completely by U^{235} alone. Further it is noted that by reason of the unusually high cross-section for fission of the rare isotope, the thermal fission process should be quite efficient.

In promulgating an efficient chain reaction the basic problem is to slow the fission produced neutrons to an energy which will produce further fissions effectively without losing the neutrons in the deceleration process.

There are several ways in which the nuclear fission process may be sustained. The most important of these for general applications is the chain reaction pile. One feature of paramount importance is that the pile can sustain the fission process with dilute fissionable materials, thus eliminating costly and time consuming separation and concentration of pure fissionable materials from natural sources. The purpose of operation of the pile may be either for power production, transformation of elements, or both.

The physical construction of the pile involves the proper arrangement of four components. The first is the fissionable material. Next is the moderator, which is a substance for controlling the speed of fission produced neutrons in such a manner that the reaction will be reproduced. Third, the auxiliary materials necessary to mechanical operation. Last, and not absolutely necessary in all cases, the coolant for removal of heat generated by the reaction. If the pile is operated at a low level of power output the heat produced is dissipated satisfactorily without assistance. Each of these basic units of pile construction will be gone into with more detail below.

On the basis of present knowledge, any element of atomic number equal or greater than 90 presents definite possibilities for use in a self-sustained fission process. This view was first advanced by Bohr and Wheeler in their consideration of nuclear stability based on the liquid drop analogy to the nucleus, given previously. The most important of the elements, however, are uranium and plutonium. These may be used in the form of purified ore (oxide); the pure element in

its naturally occurring mixture of isotopes; or the pure element with added amounts of the fissionable isotope, this last applying to the case of natural uranium with added amounts of U^{235}.

The neutrons emitted by the fission reaction possess energies on the order of 1 million electron volts. Fission absorption is most likely for neutrons with energies around 1 electron volt or less. The minimum or critical energy for fission of U^{235} is about 0.03 electron volts. Thus it is seen that the fast neutrons must be considerably decelerated for effective utilization. The most convenient way to do this is to provide a number of particles of approximately the same mass as the neutron so that the neutron will undergo a number of collisions involving a maximum loss of momentum at each impact. The particles in question are provided by the nuclei of the light elements. Thus hydrogen, helium, deuterium, beryllium, and carbon are good possibilities. The pure substance, or a suitable chemical compound of it may be used. One particularly important point that must not be overlooked in selection of a moderating material, is the possibility that it may absorb neutrons to a considerable extent and thus make them unavailable for sustaining the chain reaction. Such elements as lithium and boron are ruled out on this account.

Mechanical features of pile construction present another set of criteria for moderator selection. Helium, for instance, satisfies the basic requirements very well. Unfortunately, it is one of the permanent gases and forms no compounds. Therefore, its use is complicated by the necessity of having an absolutely gas tight container.

Deuterium and hydrogen are also good moderators, but their usual compounds vaporize at fairly low temperatures and the problem of gas tight construction is again present. Beryllium seems to be satisfactory in every way except one; at present, quantities of it in the pure form are unobtainable. Carbon in the form of graphite has been manufactured for some time in grades of relatively high purity. It was not too difficult a task to extend existing equipment and methods to produce graphite of the purity required (99.9%) for pile construction. This last, coupled with favorable physical properties, led to the use of graphite in most of the piles made in this country during the war.

The category of auxiliary materials includes the uranium casings, coolant tubes, control rods, and impurities. To facilitate handling, the uranium must be encased in some material with a very small neutron absorption tendency. In addition, the material must be one capable of easy forming and shaping. The same requirements apply to the ducts for conducting the cooling medium through the pile. Aluminum has been found to be very satisfactory for both purposes. The control rods are present to provide a means of controlling the speed of reaction in the pile. They accomplish this by absorbing neutrons that would otherwise go to promote the fission reaction. The rate of absorption is governed by pushing the rods farther into the pile or pulling them out. For this use a material with a high neutron absorption cross-section for a wide range of neutron energies is needed. Cadmium or boron steel are commonly used, although there are other possibilities such as lithium. Impurities are always

present to some extent. A great deal of effort is required to keep them at extremely low percentages. Especial attention must be paid to elimination of foreign materials with high absorption cross-sections. It may be pointed out here that the fission process produces impurities of its own by reason of the fact that most of the known elements are included in the end products of fission. Many of these have a poisoning effect on the reaction. Therefore, the effective operating time of a pile between changes of uranium rods is governed by the rate of impurity formation. To increase operating time for a given pile, the initial amount of impurities present must be reduced.

Cooling of a pile may either be carried out as such, or with the purpose in mind of utilization of the resultant heat energy. In either case, a heat transfer medium is required which will not interfere with the reaction going on. Water, and air, are both well suited to this purpose. There is also a possibility that mercury could be used. It has been suggested that bismuth, or other low melting point metals of low absorptive cross-section, could be used.

Pile Theory:

Just a few months ago Fermi's development of "Elementary Theory of the Chain-Reacting Pile" was released from security restrictions and published[12]. This section is a presentation of his basic theory together with pertinent information from other sources.

For present purposes two basic nuclear reactions are of interest. One is the absorption of a neutron by a uranium nucleus producing a critically deformed nucleus which divides into two roughly equal parts,

the fission process. The other important reaction is the absorption by the nucleus of neutrons of definite energy levels usually producing nuclei radioactively unstable with respect to beta ray transformations. An example of this last follows.

$$_{92}U^{238} + {}_{0}n^{1} \longrightarrow {}_{92}U^{239} \xrightarrow{\beta} {}_{93}Np^{239} \xrightarrow{\beta} {}_{94}Pu^{239}$$

This last type of reaction is called resonance absorption of the neutron because the nucleus will only accept neutrons with these results when the energy of the neutron is such as to be in resonance with existing mode of vibration of the nucleus. One of the most important applications of pile theory is the fact that it shows ways of minimizing the undesirable resonance absorption process, and maximizing the intensity of fission reactions.

There are three general energy intervals for neutrons, which are designated as follows. Fast neutrons are those emitted at fission with energies of a few million electron volts. Fermi limits this class to energies above the critical energy for fission of uranium 238 which is about 1.5 M.E.V. Epithermal neutrons have energies less than the critical value above and greater than thermal neutrons, whose energies are on the order of a few electron volts or less. The energies of thermal neutrons approach 3/2 kT corresponding to the energy of a particle in thermal equilibrium with its surroundings, where k is the Boltzmann gas constant per particle, 1.37×10^{-16} ergs per degree C., and T is the temperature on the absolute Kelvin scale.

The most probable occurrence when fast neutrons pass through natural uranium is that fission of U^{238} will take place. There is a smaller, but still appreciable, amount of resonance absorption. When the neutrons are in the epithermal energy range the most probable process is resonance absorption. The cross-section for resonance absorption is an irregular function of the energy of the neutron, and has a large number of absorption maxima. The resonance absorption effect becomes serious at neutron energies of less than 10 K.E.V. and increases with decreasing neutron velocity. Both fission and resonance absorptions are important phenomena in the thermal energy range. In this region the cross-sections for both reactions vary with the reciprocal of velocity in about the same way. Therefore, the relative importance of the two processes is independent of neutron energy. Fortunately, the cross-section for fission is larger than that for resonance absorption.

When a fission takes place, an average number ν of neutrons is emitted with a continuous energy distribution on the order of 1 M.E.V. If η is the average number of neutrons emitted when a thermal neutron is absorbed by uranium, then η differs from ν because only a fraction of the thermal neutrons absorbed by uranium leads to fission. A first approximation is:

$$\eta = \frac{\nu \sigma_f}{\sigma_f + \sigma_r}$$

where σ_f and σ_r are the cross-sections for fission and resonance

absorption respectively. This does not inclued leakage of neutrons from systems of finite size.

Let the discussion be limited for the present to systems of practically infinite size. If P is the probability that a fast neutron is ultimately absorbed by fission, then the average number of neutrons produced by one original neutron is:

$$k = P\nu$$

k is usually called the reproduction factor of the system. For a reaction to be self-sustaining, it is seen that k must be equal to or greater than 1. This can always be achieved if the pile is large enough and conditions on impurities are met.

The actual evaluation of the reproduction factor requires calculation of the probabilities for various events in the different energy intervals. The fundamental considerations for practical calculation follow.

The probability of fission before deceleration for a very small lump of uranium, is given by

$$P_f = \sigma_f N_u d$$

where σ_f is the average value of the fission cross-section for fast neutrons. N_u is the concentration of uranium atoms in the lump, and d is the distance that a neutron produced in the lump must travel before reaching the surface of the lump. Lump size is limited for this expression because the effects of multiple collision, elastic

and inelastic scattering become important for larger lumps. Inelastic scattering is especially important because it slows the neutrons to energies at which they are easily absorbed by the resonance process.

The great majority of neutrons are not absorbed by fast fission, but go on to lose energy rapidly due mostly to collision with carbon nuclei. It takes about 110 collisions with graphite for a neutron to be decelerated from 1 M.E.V. to 0.025 electron volt, the minimum energy for U^{235} fission. During the deceleration the neutron may be absorbed in uranium by the resonance process. For a single uranium atom in a graphite medium in the presence of decelerating fast neutrons going toward thermal energies, the probability per unit time for resonance absorption of an epithermal neutron is

$$P_r = \frac{q\lambda}{0.158} \int \sigma(E) \frac{dE}{E}$$

where q is the number of fast neutrons entering the system per unit time per unit volume; λ is the collision mean free path of these neutrons in the medium. $\sigma(E)$ is the cross-section for resonance absorption written as a function of neutron energy. This is necessary because the resonance absorption phenomenon is not continuous with energy distribution, but takes place at a number of specific energies. The limits of integration are from just above thermal energy to the average energy of fission neutrons. This formula is not applicable to lumps of uranium, because of a very important self-screening effect which reduces the neutron flux inside the lump. Therefore, the practical solution to

the problem is a direct measurement of neutron absorption in uranium lumps of various sizes. These measurements have been made, but are not available for publication.

Graphite has an appreciable absorption cross-section for thermal neutrons. Therefore, its effect may no longer be ignored. If a uniform mixture of carbon and uranium is considered, a thermal neutron will have a probability of absorption by uranium of:

$$P_t = N_u \sigma_u / (N_c \sigma_c N_u \sigma_u)$$

where N_c and N_u are the numbers of carbon and uranium atoms per unit volume, and σ_c and σ_u are the absorption cross-sections for thermal neutrons of carbon and uranium.

The density of thermal neutrons in a lattice distribution of uranium in graphite is not constant. Within a lump of uranium the thermal neutrons are rapidly absorbed, thereby reducing the number available. In addition, the density of uranium and carbon atoms varies as a function of the three space coordinates. Therefore, the solution for a non-homogeneous distribution of uranium in a moderator is considerably more complicated. An approximate result may be obtained by introducing the average densities of thermal neutrons in graphite, and uranium lumps, n_c and n_u. The number of neutrons absorbed by each material will be proportional to the product of the density of nuclei, the density of neutrons, and the absorptive cross-section. Then the following may be written:

$$P_t = \frac{N_u \sigma_u n_u}{N_u \sigma_u n_u + N_c \sigma_c n_c}$$

The average neutron densities are either measured, or estimated by use of diffusion theory. In the latter case, an approximation is made of substituting a spherical cell of equal volume for the lattice cell and applying the boundary condition that the radial derivative of neutron density vanishes at the surface of the cell. It is also assumed that the number of neutrons decelerated to thermal energies per unit time and unit volume is constant in the graphite part of the cell. By using these approximations Fermi obtained an equation, applicable to small piles, for the probability of thermal neutron absorption by uranium of:

$$P_t = \frac{3\alpha^2(1-\alpha)(1+\beta)e^{-\beta+\alpha} - (1+\alpha)(1-\beta)e^{\beta-\alpha}}{(\alpha^3-\beta^3)(\alpha+s-s\alpha)(1+\beta)e^{-\beta+\alpha} - (\alpha+s+s\alpha)e^{\beta-\alpha}}$$

where α and β are the radii of the lump and the cell respectively, using the diffusion length in graphite as the unit of length.

$$\text{unit length} = \sqrt{\lambda\Lambda/3}$$

and

$$s = \frac{\lambda(1+r)}{\sqrt{3}(1-r)}$$

r is the reflection coefficient of the lump for thermal neutrons,

λ is the collision mean free path for thermal neutrons in uranium,

Λ is the absorption mean free path.

One of the most important factors in pile design is to minimize the resonance absorption of neutrons by uranium during the deceleration period. In an homogeneous pile this effect may be reduced by decreasing

the amount of uranium present. On the other hand, when the neutrons have reached thermal velocities the probability of absorption by uranium is in the ratio of the absorptive cross-sections of carbon and uranium multiplied by the concentration of each element. Decreasing the uranium content of the pile thus decreases the possibility of ultimate fission absorption. Therefore, as is usual in design, two opposite requirements must be balanced to obtain an optimum system.

If design is not restricted to homogeneous mixtures a more favorable situation may be obtained by the proper distribution of the fissionable material and the moderator. This is possible because of the following reasons. The resonance absorption of neutrons during deceleration has very sharp cross-section maxima. If the uranium is concentrated in the form of lumps, the nuclei in the interior will be shielded from neutrons close to an absorption maximum by the thin outer layer of uranium. The neutrons hitting the outer layer of uranium will either be absorbed without fission (epithermal neutrons), decelerated considerably by inelastic collision, or deflected with little loss in energy by elastic collision. In this way the interior nuclei are subjected to a much less intense stream of epithermal neutrons and, therefore, the overall probability for resonance absorption is reduced. a counter-balancing factor is that lump distribution also reduces the thermal neutron density inside the lump.

It can be expected theoretically, and has been shown, that up to a certain size of lump the gain from reducing resonance loss overbalances considerably the loss by lesser absorption in the thermal range.

In actual practice, typical pile structure is a lattice of uranium lumps in a graphite medium. The cubic or spheroidal lump is the most efficient form, but the more commonly used is the long, cylindrical rod passing through the pile. The latter arrangement is used because it has decided practical advantages. First and foremost is that it is much easier to slide the rods out of the pile than it is to dismantle the pile to get at the lumps distributed in the interior. The operation must be done by machinery in either case, and has to be done fairly frequently to decontaminate the uranium. The rod type construction also permits a more favorable heat removal situation by permitting the coolant to be circulated only about the sources of heat production, the masses of fissionable material.

A table from data given by Fermi is shown below to give some typical figures for the probabilities of the various absorption processes. The values are averages for a good lattice.

Probability (%)	Type Process	Neutrons produced per neutron absorbed	Neutrons per generation by one neutron
3	Fast Fission	2	0.06
10	Epithermal Resonance Absorption	0	0
10	Absorption by Carbon	0	0
77	Uranium Thermal Absorption	η	0.77 η

where η, as defined above, is the average number of neutrons emitted when a thermal neutron is absorbed by uranium. The reproduction factor for this example will be:

$$k = 0.06 + 0.77\,\eta$$

η must be greater than 1.22 for this pile to have a reproduction factor greater than one.

In a lattice containing a large number of cells, the density of neutrons of a particular energy is a function of the position in the lattice. A first approximation description of such a system may be made by substituting an equivalent homogeneous form instead of the actual non-homogeneous one. The problem is then simplified by substituting average actual values for neutron densities over the volume of the cell.

Let $Q(x,y,z)$ be the number of fast neutrons produced per unit time and volume at each position in the lattice. Let $q(x,y,z)$ be the number of thermal neutrons produced, by deceleration, per unit time and volume at the position x,y,z. "q" is called the "density of the nascent thermal neutrons". The assumption is made that if a fast neutron is generated at a point, the probability of its becoming thermal at a given place has a uniform distribution about the point of origin. Justification for this is that the process of deceleration consists of a great number of free paths, which should give an average net distance travelled during deceleration to thermal velocities. It has been found experimentally that the distribution curve of

nascent thermal neutrons around a point source is more closely given by several Gaussian curves with different ranges taken in the composite. The actual formulae used are not available.

Let each fast neutron produce p thermal neutrons. For a point source of thermal neutrons at the coordinate origin, with a strength 1, Fermi gives the distribution as:

$$q_1 = pe^{-r^2/r_0^2} / \pi^{3/2} r_0^3$$

The range, r_0, is about 35 cm. for graphite of density 1.6. The density of nascent thermal neutrons at point P can be expressed in terms of Q by summing all contributing infinitesimal sources, $Q(P')dr'$, where dr' is the element of volume around P'. An expression may then be found as follows:

$$q(P) = p / \pi^{3/2} r_0^3 \int Q(P') e^{-2r'(P'-P)^2 / r_0^2}$$

The density of the thermal neutrons, $n(x,y,z)$, is related to q by the equation:

$$(\lambda v/3) \Delta n - vn/\Lambda + q = 0$$

λ is the collision mean free path of thermal neutrons, v is their velocity, and Λ is the mean free path for absorption in the mixture. The first term represents the increase in neutrons due to diffusion;

the second, the loss of neutrons due to absorption; and the third, the effect of the nascent thermal neutrons.

It was shown previously that the absorption of thermal neutrons in a lattice is due mostly to uranium. The absorption mean free path, Λ, in the above equation is much less than the value for pure graphite, Λ_0. The following relationship holds as a first approximation:

$$\Lambda = (1 - P_t)\,\Lambda_0$$

Actual values for uranium and graphite are on the order of 300 cm., and 2,500 cm. respectively.

An average number ν of fission neutrons is produced per thermal neutron absorbed by uranium. The total number of new neutrons is slightly greater because of the small probability of fast fission. Let $\epsilon\eta$ be the total number of fast neutrons produced. The number of thermal neutrons absorbed per unit itme and volume is vn/Λ. The fraction P_t of this number is absorbed by uranium. Then the number of fast neutrons produced is:

$$Q = P_t\,\epsilon\eta\nu n/\Lambda + Q_0$$

The second term is the number of fast neutrons from any outside source, usually zero.

Taking the above fundamental equations as a basis and imposing restrictions found from experimental evidence, Fermi gives the

reproduction factor for a pile of infinite size as:

$$\left(1 + \frac{\pi^2 \lambda \Lambda}{a^2}\right) e^{3\pi^2/a^2 r_0^2/4} = \epsilon \eta p P_t = k_\infty$$

In an actual pile the reproduction factor is close to that for an infinite pile because the lattice is covered with a neutron reflecting substance which reduces leakage to a great extent. Thus he further simplifies the formula by experimental considerations to give:

$$k = 1 + (3\pi^2/a^2)(\lambda \Lambda/3 + r_0^2/4)$$

The critical side length "a" of a cubical pile may be calculated from this. Assume values for a special lattice of $\lambda = 2.6$ cm., $\Lambda = 350$ cm., $r_0^2 = 1200$ cm.^{2}, $k = 1.06$. Then the critical side is 584 cm., and the volume 199 cubic meters. The values assumed are purely hypothetical, but give an idea of the order of magnitude of actual cases. Practical cases are strongly dependent on details of lattice structure.

An approximate relationship can be obtained between the power produced by a pile and the intensity of thermal neutrons inside it. Roughly 50% of the neutrons produced in a pile lead to fission. The energy release per fission is about 190 M.E.V. Then the output is about 1.52×10^{-4} ergs per neutron absorbed. The number of thermal neutrons absorbed per unit volume is vn/Λ. Then the approximate energy output is:

$$(vn/\Lambda)\ 1.52 \times 10^{-4} = 4.3 \times 10^{-7}\ vn\ \text{ergs/cm.}^3\ \text{sec.}$$

assuming Λ is 350 cm.

Power is not produced uniformly throughout a pile because n is a maximum at the center and decreases to zero at the edge. For any point in a pile, n is given by Fermi as a function of the space coordinates x,y,z; n_o, the density of neutrons at the center, and "a", the critical length of a side.

$$n = n_o \sin(\pi x/a)\sin(\pi y/a)\sin(\pi z/a)$$

From this, by integrating over all the volume of the pile, he obtains and expression for power:

$$W = (8/\pi^3)\ 4.3 \times 10^{-7}\ n_o v a^3 = 1.1 \times 10^{-7} n_o v a^3$$

For a critical side of 584 cm., as given in the example, the power is $W = 22\ n_o v$ ergs/sec. If the operating power is about 1 kilowatt, the flux of thermal neutrons at the center is approximately

$$n_o v = 4 \times 10^8\ \text{neutrons/cm.}^2\ \text{sec.}$$

There are three fundamental types of jet engines. They are the pulse-jet, as exemplified in the German V-1; the turbo-jet, made famous by the Army's P-80 Shooting Star, and the ram-jet, whose first successful public try-out was given by the Navy's Bumble-Bee project in the summer of 1946. The pulse-jet depends for its operation on an intermittent burning process with pulsating production of thrust. This type does not seem compatible with the steady energy production of a nuclear fission source of power. The ram-jet and turbo-jet, however, require a steady, high rate of heat supply. This would seem to make these last two engines well suited for the application of the tremendous amounts of energy that can be released by the fission process.

In connection with the chain reaction pile discussed previously, the possibility of another type of nuclear reactor should be mentioned. This is the idea of having a greater than critical mass of some pure fissionable material and conducting what might be called a "controlled atomic bomb reaction". The control would be furnished by having control rods, similar to those in the pile, placed in the U^{235} or plutonium. This would eliminate the tremendous weight, around 100 tons, involved in the use of an ordinary pile. At first glance this seems like a reasonable application for military purposes. The cost alone, at the present time, of a large amount of pure fissionable material would make commercial use prohibitive. There is one very serious shortcoming to this scheme, namely, the production of a mechanism that could actuate the control rods properly

in a matter of milli-seconds or less.

The only known method of control of fission reactions is the modification of neutron density by absorbers such as cadmium. This involves variation of the physical depth of penetration of the control substance in the reactor. In order to produce power, the reproduction factor of the reactor must be at least unity. As soon as the reproduction factor becomes very slightly greater than one for a brief interval, an atomic explosion results. Even if a control mechanism were developed that could receive information, monitor it, and actuate the control rods in the small amount of time allowed, the very slightest malfunctioning of the mechanism would result in an explosion of the size that devastated Hiroshima.

It is commonly stated that U^{235} fission tends to be thermally stable. This is based on experimental evidence that the cross-section for capture of neutrons decreases with increasing temperature of uranium and with increasing neutron velocity. In a reaction of the type under consideration, the fission process is initiated by fast neutrons and the temperature of the uranium is already high for normal operation. Under these conditions the first approximation relationship for cross-section variation no longer holds completely. Furthermore, a reduced cross-section merely means that the neutrons have greater opportunity for absorption by other substances, such as are present in a pile. In the case of pure fissionable material, the neutrons will still be absorbed by the fission process.

From the above considerations, the writer believes that nuclear fission power sources of low weight near the critical mass of fissionable material are not likely in the near future. However, there is still a very important possibility to consider, that of utilization of a chain reaction pile type of heat source.

It has been shown at Hanford and other places that the chain reaction pile can be operated effectively and with safety. These units employ uranium in its naturally occurring mixture of isotopes. A more effective arrangement for power applications where weight is a major consideration could be obtained by using natural uranium enriched with added U^{235}. A probable maximum of U^{235} for safe operation might be on the order of 10% without risk of explosion.[13] Thus the overall weight might be reduced to something in the neighborhood of 10-15 tons, proportional to the 100 ton approximation given for the first pile built in Chicago, which contained about 6 1/4 tons of natural uranium metal. This weight would be increased, however, when air-heating ducts were inserted in the pile, thus reducing the effectiveness of the pile by wider separation of the elements of the chain-reaction lattice.

The ducts for heating the air pose another limitation on the pile with respect to its utilization in a jet-engine. Even if the frontal cross-section of the pile presents as much as 50% of its area for air passage, the frontal area of the engine and thus the drag will be increased considerably compared to combustion powered engines.

This must be thought of in connection with other considerations. At the present time, the operation of chain-reaction piles is restricted to low temperatures. Limitations are the reduction of fission cross-section with increasing temperature in the presence of thermal neutrons, and the physical properties of materials of which the pile is composed. Aluminum is the only published or suggested material which fills the qualifications for coolant tubes and uranium casings. Even if a suitable higher melting point material is found, the decreased cross-section for fission at higher temperatures will necessitate an increase in the critical size of the pile. Thus it is apparent that several years at least will be required before pile operating temperatures begin to compare with temperatures now easily available with combustion. Furthermore, given a pile that could be operated at high temperatures, the probable weight increase seems to remove the pile from consideration in the near future for the sizes of guided missiles now envisioned.

These weight considerations have not included any consideration of the amount of shielding that may be necessary. For an uninhabited missile the shielding requirements would be limited to whatever might be necessary to prevent interference with electronic equipment. For a craft carrying passengers, the requirements become much more stringent. No specific information is available on shielding requirements for neutron emanations. Neutrons, due to their lack of charge, are very penetrating and it is known that the thickness of shielding material needed is on the order of several feet of concrete and water.

The picture presented so far may seem to indicate that there is no hope for application of atomic power to any but tremendous sized aircraft by present standards. This is not believed by the writer to be the case. If utilization is restricted purely to the fission

process, there will probably be a lapse of at least ten years before it will be possible to revise accurately present estimates of utility of nuclear fission power applications. The prospect of the large energy releases possible from nuclear reactions is much too attractive to let pass without further investigation of other possibilities, however.

An idea advanced by Gamow and others appears to be particularly interesting in the light of the above. This suggestion is to use radioactive elements as a source of power. The power to be derived from that given off in the radioactive decompostion process. Actually this can be considered as an indirect use of fission because the radioactive material used would come from the operation of a pile and the extraction of radioactive fission products, or the irradiation of non-radioactive isotopes to get induced radioactivity. The chain reaction pile provides the first practical source of radioactive isotopes in large quantities.

The type of isotope desired would be one with a fairly short half-life of a few days, so that there would be a reasonably rapid but uniform release of energy. Consider as an example the radioactive isotope $_{31}Ga^{74}$. It decays emitting a beta particle with 0.8 M.E.V. of energy and has a half-life of 9 days. If five pounds, containing about 1.84×10^{25} atoms, is taken as an energy source, the average rate of energy release for the first nine days will be two hundred horsepower. This is based on the fact that half of the atoms present at the beginning will undergo decomposition with each releasing 0.8 M.E.V. A more desirable type of reaction might be

one in which alpha particles (helium nuclei) are produced. The alpha particles have a shorter range and therefore give up their energy in a smaller thickness of the medium being acted upon. This also makes the shielding problem much easier.

It must be remembered that while radioactive energy sources present no danger of explosion, the energy release is a maximum when the material is first placed in the engine and decreases with time. This might be overcome by using isotopes which yield further radioactive materials with similar characteristics, so that the reduction in output of the original material would be balanced by activity of the product material as the amount of product increased with time. This balance would be only approximate for a simple combination, because the secondary material would reach a peak concentration and then fall off by reason of its own activity and its reduced rate of formation from the primary source. It is conceivable, however, that mixtures of primary materials could be devised that would provide a reasonably steady energy release for desired periods of time.

For jet propulsion applications the radioactive release of energy could be used in the form of heat caused by the absorption of the emanations in an air medium. The radioactive material might be coated on the walls of what is now the combustion chamber, or used in the form of slugs of any desired size or shape. The application to either ram-jets or turbo-jets seems reasonably straightforward. From a first inspection of the problem, the development seems to depend primarily on selection of suitable materials. The transfer of radioactive energy to air is accompanied by heavy ionization. This must be considered with respect to increased corrosion of structural parts.

The writer has endeavored to make a forecast of the likely modes of application of atomic power to jet engines, especially those intended for use in guided missiles. This forecast is based on as careful a study as time permitted of the known and available information on nuclear fission and related subjects. The prognostications of others have been consulted on the basis of the information acquired. Briefly, the conclusion reached is that nuclear fission as such does not present good prospects for utilization in guided missiles in the near future. Application to large aircraft seems to be of greater likelihood. Apparently a much more immediate possibility for use of atomic power in jet engines for guided missiles is presented by radioactive sources of energy. It is believed that this latter type of atomic power source could be applied to existing basic types of jet propulsion engines. The writer regrets that limitations of time prevent a more extensive survey of possibilities in this field.

For further work on the subject it is suggested that a thorough study of radioactive phenomena and materials be made with the above use in mind. Study should also be given to find the effect of various radioactive phenomena on structural materials, and to possible limitations imposed.

1. E. Fermi, Proceedings of the Royal Society, v. 149, p.522, 1935.

2. Pool, Cork, & Thornton, Physical Review, v.52, p.239, 1937.

3. Cork & Middleton, Physical Review, v.58, p.474, 1940.

4. E. Fermi, Nature, v.133, p.898, 1934.

5. M. Henderson, Physical Review, v.57, p.774, 1940.

6. J. M. Cork, Radioactivity and Nuclear Physics, Edwards Brothers, Inc., Ann Arbor, Mich., 1946.

7. Roberts, Hafstad, Meyer, Wang, Physical Review, v.55, p.664, 1939.

8. Bohr, Wheeler, Physical Review, v.56, p.426, 1939.

9. Feenberg, Physical Review, v.55, p.504, 1939.

10. Present, Reines, Knipp, Physical Review, v.70, p.557, 1946.

11. Nier, Booth, Dunning, Grosse, Physical Review, v.57, p.546, 1940.

12. E. Fermi, Science, v. 105, #2715, Jan. 10, 1947, p.27.

13. C. Goodman, Automotive and Aviation Industries, Dec. 1, 1945, p.18.

14. Aviation, Sept. 1945, v.44, #9, p.103.

15. Science News Letter, v. 49, pp. 212, 228, 248.

16. Science News Letter, v. 50, pp. 85, 179.

17. C. H. Thomas, Chemical & Engineering News, v.24, p.2480, Sept.25,1946.

18. H.D. Smyth, Atomic Energy for Military Purposes, Princeton Univ. Press, Princeton, N.J., 1945.

19. Smith, Fox, Sawyer, & Austin, Applied Atomic Power, Prentice-Hall Inc., New York, N.Y., 1946

20. G. Gamow, Atomic Energy In Cosmic and Human Life, Macmillan Co., New York, N.Y., 1946.

21. J. D. Stranathan, The "Particles" of Modern Physics, The Blakiston Co., Phila., Pa., 1946.

22. A. K. Solomon, Why Smash Atoms?, Harvard Univ. Press, Cambridge, Mass., 1940.

23. N. Bohr, Atomic Theory and the Description of Matter, Macmillan Co., New York, N.Y., 1934.

24. A. M. Weinberg, Physical Review, V. 70, p.115, 1946.

25. Fermi & Zinn, Physical Review, v. 70, p.103, 1946.

26. Burgy, Pardue, Willard, Wollan, Physical Review, v.70, p.104, 1946.

27. L. W. Nordheim, Physical Review, v.70, p.115, 1946.

28. H. Soodak, Physical Review, v.70, p.115, 1946.

29. Kennedy, Seaborg, Segré, & Wahl, Physical Review, v.70, p.555, 1946.

30. Way & Wigner, Physical Review, v.70, p.115, 1946.

AUG 31
OCT 13
OCT 27
DEC 9
DEC 20

BINDERY
4478
4478
4404
4565

Thesis 7867
R55 Rimer
AUTHOR
Nuclear fission and its applica-
TITLE tion to jet propulsion engines

DATE LOANED	BORROWER'S NAME	DATE RETURNED
7 Mar'49	E.C. MILDAHN	3 Mar'49
21 MR 51	R C Morris	✓
[illegible]	J Hapon	A24
MAY 29 52	Holderman	A26

Thesis 7867
R55 Rimer
Nuclear fission and its application to jet propulsion engines.

Library
U. S. Naval Postgraduate School
Monterey, California

BRIGHT GOLD LETTERING
SMITH BINDERY
OAKLAND, CALIF.

thesR55
Nuclear fission and its implication to j
3 2768 001 91338 7
DUDLEY KNOX LIBRARY